Australian Journal of Teacher Education

Volume 38 | Issue 11 Article 4

2013

What Teachers Need to Know to Teach Mathematics: An argument for a reconceptualised model

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Abstract: Since Shulman's (1986) seminal work on Pedagogical Content Knowledge (PCK) was released, it has created opportunities for the creation of constructs to scaffold the knowledge and understandings that teachers need in order to be effective. Adapting this work from being a heuristic to an operational structure has seen the development of many models. One such model regarding Mathematical Knowledge for Teaching (MKT) (Hill et al., 2008) has enjoyed a good deal of attention. This paper aims to argue for a reconceptualisation of Hill et al.'s (2008) model to make it as informative as possible for teachers and teaching.

When Australia fails to improve or maintain its position in the international testing (Trends in International Mathematics and Science Study [TIMSS] or Programme for International Student Assessment [PISA]) in the subject of mathematics, there is an outcry. Headlines such as "Aussie schools flatline in global education tests" (Australian Broadcasting Corporation, 2012) appear on the national broadcaster's website as a news item. Recriminations are loud and some media, politicians and academics lament the standard of teaching (Aylmer, 2013; Dinham, Ingvarson & Klienhenz, 2008). In identifying quality of teaching as one of many factors which can contribute to determining mathematical achievement throughout the country, this may in some instances be warranted. However, identifying a problem and providing a solution for that problem are two widely different things. The usual response of developing curriculum and articulating required standards (as has happened in recent years in Australian education) is important, but as Ball, Hill and Bass (2005) state "…little improvement is possible without direct attention to the practice of teaching" (p.14). It may seem almost self-evident that if society requires effective learning, then effective teaching is necessary. The questions which need to be addressed are; what constitutes the professional knowledge required for teaching mathematics effectively and can it be represented in a well-considered heuristic.

Shulman's (1987) Pedagogical Content Knowledge (PCK)

In his seminal work, Shulman (1987) specified seven categories of professional knowledge required for teaching. These categories are: content knowledge; general pedagogical knowledge; curricular knowledge; knowledge of learners (their characteristics, cognition, motivation and development); knowledge of educational contexts; knowledge of educational aims, goals and purposes; and pedagogical content knowledge (PCK). The category which caused the greatest excitement amongst researchers was PCK (Abd-El-Khalick, 2006; Ball, Thames & Phelps, 2008).

Shulman (1986) argued the existence of PCK, a form of knowledge building upon, but not the same as, subject matter knowledge or knowledge of general principles of pedagogy.

Rather, the epistemological idea of PCK could be described as a link between the knowledge bases of content and pedagogy with a third necessary domain of context (Figure 1). One could characterise PCK as a practical knowledge of teaching and learning guided through a contextualised knowledge of a particular classroom setting.

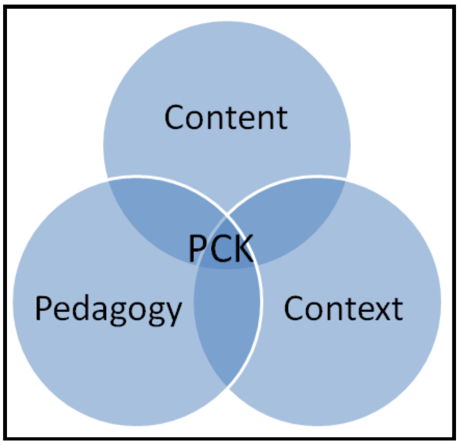

Figure 1: Shulman's (1986) domains of pedagogical content knowledge

Park and Oliver (2008) reviewed and analysed literature available on defining PCK and arrived at what they considered to be a comprehensive working definition:

> PCK is teachers' understanding and enactment of how to help a group of students understand specific subject matter using multiple instructional strategies, representations, and assessments while working within the contextual, cultural, and social limitations in the learning environment. (p. 264)

There is research to support that beginning teachers possess a limited repertoire of PCK (Lee, Brown, Luft and Roehrig, 2007; Nason, Chalmers & Yeh, 2012; Wilson, Floden & Ferrini-Mundy, 2002) and that experience is a major influence on the shaping and development of a teacher's PCK (Kleickmann et al., 2013; Lee et al., 2007). There is further evidence to support that teaching experience alone is not sufficient and that experience, coupled with thoughtful reflection of instructional practices is required (Kleickmann et al., 2013). This thoughtful reflection may help mitigate the findings of Schneider and Plasman (2007) who warn that PCK might actually decline over time.

Gess-Newsome and Lederman (1995) also concluded that teaching experience, although an important factor in the development of PCK, is not as significant in contributing to PCK as a teacher's opportunity and disposition towards reflection on content knowledge. They state:

> …teaching experience alone does not equate with teaching expertise, though the two are often mistakenly confused. Opportunities for a teacher to reflect on classroom practice and implement identified changes, however, greatly influenced teaching "expertise". If teaching is to be a purposeful act, and if we want teachers to be able to translate integrated understandings of content into classroom practice, the time and opportunity to develop, codify, and implement such beliefs into the classroom must be fostered… (p. 321).

Refinements of Pedagogical Content Knowledge (PCK)

In 1999 Gess-Newsome proposed two models for PCK, integrative and transformative. The integrative model supposes that the relevant knowledge bases used in teaching (Shulman's domains, 1986) are developed separately and that the act of teaching provides opportunity for their integration. Therefore "an expert teacher, then, is one with organised knowledge bases that can be quickly and easily drawn upon while being engaged in the act of teaching" (Silverman & Thompson, 2008, p. 501).

The transformative model is a model which recognises the value of synthesised knowledge, the fundamental transformation of knowledge and the creation of new knowledge. According to Silverman and Thompson (2008) the transformative model requires purposefully integrated experiences that allow teachers the opportunity to not only extend their mathematical and pedagogical understandings but also to create connections to create a new knowledge. The transformative model recognises that whilst the knowledge bases of content, pedagogy and context exist they are only useful when transformed into PCK, which by extrapolation cannot occur in a professional learning situation or in pre-service training, but only in the classroom. Is there then an argument to be made that unless teachers are sensitised to the elements that are required for PCK to develop, that they will have issues in creating the required connections whilst in the busy environment of their classroom?

Questions need to be asked regarding the efficacy of the current manner in which systems and sectors try to enhance the PCK of teachers and teaching neophytes. To become more responsive when the opportunities for development of PCK in the teaching work place present themselves, teachers need to be given purposeful development opportunities to reflect upon their teaching. This may take the form of having opportunities to observe, analyse and reflect upon other teachers' teaching (Nilsson, 2008). Furthermore, these other teachers, whether they are classroom based or in a university setting, may in the words of Nilsson (2008) "…need to portray and explicate aspects of their own PCK…" (p. 1296).

Developing PCK for teaching mathematics

It is well documented that many teachers exhibit weaknesses and lack a deep conceptual understanding of mathematics (Ball, Hill & Bass, 2005; Hill et al., 2008; Ma, 1999;Tsao, 2005). There have also been numerous studies which support the contention that many teachers lack confidence and content knowledge when teaching mathematics (Ambrose, 2004; Evans, 2011; Hill et al., 2008; Kajander, 2005; Norton, 2010; Tsao, 2005). This is supported by Askew (2008) who states that; "…one thing is clear from the research evidence: many prospective and practising primary teachers have, or express, a lack of confidence in their mathematical knowledge" (p. 16). This is a worrying finding, as a positive attitude towards the teaching of mathematics has been shown to have a direct influence on the levels of success that teachers can help students to achieve (Kulm, 1980; Sullivan, 1987). Given this lack of confidence and content knowledge it seems even more important that teachers have some sort of framework upon which to base the development of PCK.

Regardless of its pre-eminence, Shulman's notion of PCK has been challenged (Graeber & Tirosh, 2008) and the concept has been expanded and modified by a number of authors. Ball and Bass (2000) for example, regard the ability to unpack the mathematics from constructs, concepts, analogies, metaphors and images as being an important aspect of PCK.

Mathematical Knowledge for Teaching (MKT), Hill, Ball and Schilling (2008)

Ball, Thames and Phelps (2008) argue that although the term PCK is very widely used, it actually lacks clarity of definition, and its potential has not been fully realised, a view shared by Schneider and Plasman (2011). Their refinements of the concept of PCK and its attempt to reframe the study of teaching knowledge are predicated around placing the emphasis on the use of "knowledge in and for teaching rather than on teachers themselves." (p. 394). In elaborating on Shulman's construct of PCK, several research teams (Hill, Ball & Schilling, 2008; Ball, Thames & Phelps, 2008; Delaney, Ball, Hill, Schilling & Zopf, 2008) use a construct (Figure 2) which maps their domains of content knowledge for teaching onto two of Shulman's (1986) initial categories for PCK, those of subject matter knowledge and pedagogical content knowledge. Hill et al.'s (2008) construct of Mathematical Knowledge for Teaching (MKT) has been described by Depaepe, Verschaffel and Kelchtermans (2013) as the most influential reconceptualisation of teachers' PCK within mathematics education.

Depaepe et al. (2013) cite three clear merits of MKT: that it was borne out of empirical research on the knowledge teachers require to teach mathematics; that MKT took Shulman's (1986) heuristic and turned it into a valid measure of teachers' mathematical knowledge for teaching; and lastly, that it provides empirical evidence of a positive relationship between student learning and teachers' PCK.

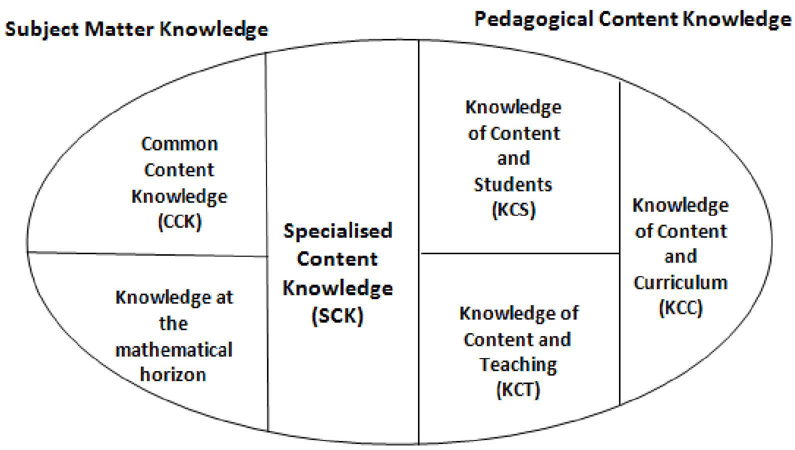

Figure 2 Mathematical knowledge for teaching (Hill et al., 2008, p. 377)

Common Content Knowledge (CCK) is mathematical knowledge and skill used in general settings, settings not necessarily unique to teaching. Specialised Content Knowledge (SCK) is the mathematical skills and knowledge particular to teaching. Knowledge of Content and Students (KCS) is that knowledge which is a combination of knowing about students and about mathematics, and Knowledge of Content and Teaching (KCT) is a combination of knowing about teaching and about mathematics. The final two domains Knowledge at the mathematical horizon and Knowledge of Content and Curriculum (KCC) are both considered by the Hill et al. (2008) as interim placements, still in need of revision and refinement, as both may run across several categories or be categories on their own (Ball, Thames & Phelps, 2008).

Ball, Hill and Bass (2005) state;

> Until and unless we, as educators, are willing to claim that there is professional knowledge that matters for the quality of instruction and can back that claim with evidence, we will continue to be no more than one voice among many competing to assert what teachers should know and how they might learn that, and why (p. 45).

Once these domains of professional knowledge are accepted by educators as being necessary, it is possible to frame some supporting questions which may be used to audit where professional development might be appropriate. Some examples of such questions are provided in Table 1.

Domain	**Examples**. Are you able to:
Common Content Knowledge (CCK)	• calculate an answer correctly? • solve mathematical problems correctly? • understand the mathematics you teach? • recognise when a student gives a wrong answer? • recognise when a text book is inaccurate or gives an inaccurate definition? • use terms and notations correctly?
Specialised Content Knowledge (SCK)	• present mathematical ideas? • respond to students' why questions? • find an example to make a specific mathematical point? • recognise what is involved in using a particular representation? • link representations to underlying ideas and to other representations? • connect a topic being taught to topics from prior or future years? • explain mathematical goals and purposes to parents? • appraise and adapt the mathematical content of textbooks? • modify tasks to be either easier or harder? • evaluate the plausibility of students' claims? • give or evaluate mathematical explanations? • choose and develop useable definitions? • use mathematical notation and language and critiquing its use? • ask productive mathematical questions? • select representations for particular purposes?
Knowledge at the mathematical horizon	• make connections across the topics in mathematics? • make connections between the different strands in mathematics? • articulate how the mathematics you teach fits into the mathematics which comes later?
Knowledge of Content and Students (KCS)	• anticipate what students are likely to think? • predict what students will find interesting and motivating when choosing an example? • anticipate what a student will find difficult and easy when completing a task? • hear and interpret students' emerging and incomplete ideas? • recognise and articulate misconceptions students carry about particular mathematics content?
Knowledge of Content and Teaching (KCT)	• sequence mathematical content? • select examples to take students deeper into mathematical content? • select appropriate representations to illustrate the content?
Knowledge of Content and Curriculum (KCC)	• articulate the strands in the curriculum? • articulate the proficiencies from the mathematics curriculum? • articulate a familiarity with the structure of the mathematics curriculum?

Table 1: Domains of Knowledge and supporting questions as adapted from Ball, Thames & Phelps (2008)

This writer was involved over a period of six years in working with practising teachers in providing support in their mathematics classrooms and in the provision of professional learning (PL) opportunities. Over this six year period a few teachers expressed a passing knowledge of the work of Hattie (2005) on the dimensions of an expert teacher, fewer indicated they had heard of Shulman (1986) and his work with PCK and none at all indicated an awareness of the work of Ball, Thames and Phelps (2008). When asked about their personal criteria for effective teaching, none based their criteria around any of these studies, nor other research findings. Whilst in no way trying to claim these informal observations as being representative of the total teaching population they do warrant pause for consideration. They raise the issue that if teachers (or administrators for that matter) wish to reflect on the effectiveness of their mathematics teaching, against what can they scaffold their judgements?

Reservations Regarding Hill et al.'s (2008) model

There are some concerns regarding Hill et al.'s model. The first reservation is that the 'line' between the Common Content Knowledge (CCK) and the Specialised Content Knowledge (SCK) is blurred. This blurring is a concern shared by Thanheiser, Browning, Moss, Watanabe and Garza-Kling (2010). Yet while this is problematic in an analytical and academic way, in a practical sense it seems to make little difference where one domain finishes (CCK) and the other one begins (SCK). This issue is acknowledged by Ball et al. (2008) who refer to this as being a 'boundary' problem. They state that it is not always easy to discern where one domain divides from the next.

The second issue is simply in the representation of the domains in the diagram and the possibility of using the visual cue of region size to determine the importance of one domain of knowledge over another. In other words, because SCK appears to employ a larger area (Figure 2) is it therefore, perhaps more important than KCC? This writer could find no evidence in any reading, that in any other than specific circumstances, any one domain was necessarily more important than any other.

The third reservation is in using the term Pedagogical Content Knowledge (PCK) to describe the domains regarding pedagogical concerns. Perhaps Pedagogical Knowledge (PK) may have been a better term to employ as there is a strong argument to be stated that PCK actually only occurs at the overlap between the SMK and PK. As stated earlier, PCK is actually the link between the knowledge bases of content, pedagogy and context. The fourth and final reservation is in the fact that the model does not display the possibilities of all of the interactions between the domains.

Considerations for Refinements of Hill et al.'s (2008) Model

Due to these four small reservations, this writer offers for consideration some refinements to Hill et al.'s 2008 Mathematical Knowledge for Teaching model (Figure 2). In this revised model for Mathematical Knowledge for Teaching (MKT), each of the domains of MKT have been illustrated by regions which are the same size. This is indicative that until a specific circumstance has been determined no domain is fundamentally more important than any other domain. Indeed it will be circumstance which determines which domain or domains have priority.

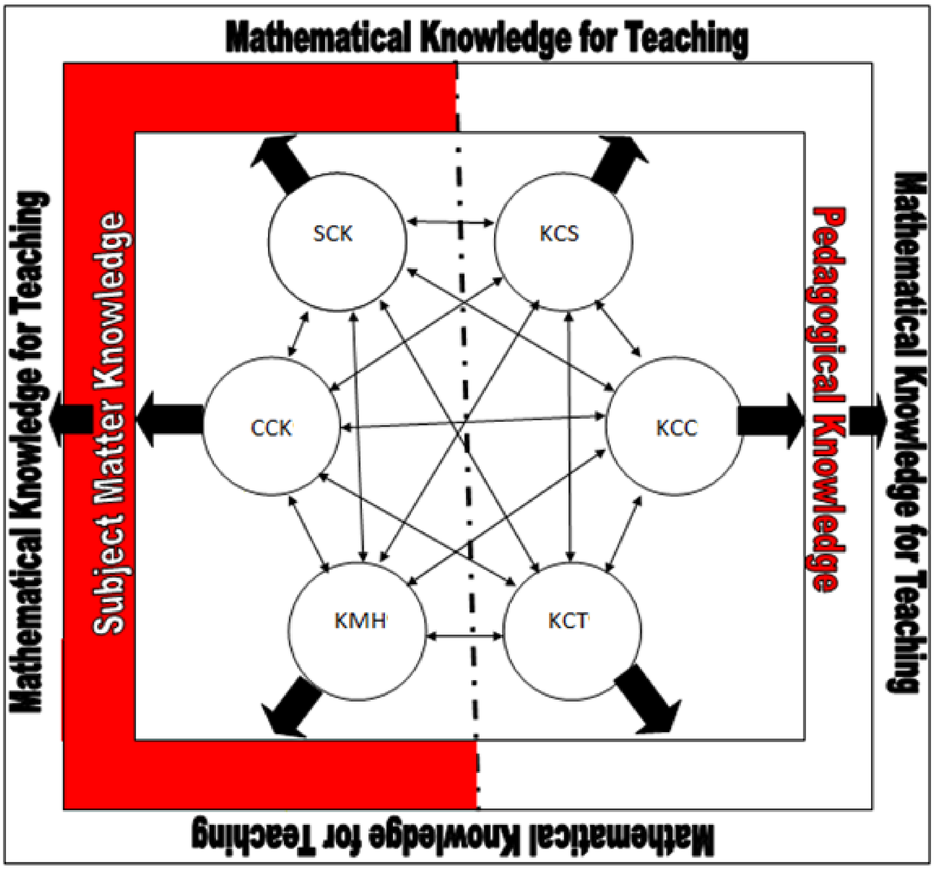

Figure 3 Revised model for Mathematical Knowledge for Teaching

For example, with the introduction of the Australian Curriculum (the first time a national curriculum has been introduced across all states and territories in Australia) many PL opportunities are being constructed regarding improving teachers' familiarity with the associated documents, what they are required to teach and what the students are required to learn. It can be argued that in a context where the content within a strand (for example Number and Algebra) is well known to teachers then the PL can afford to be more focussed on the Pedagogical Knowledge (PK) concerns; Knowledge of Content and Teaching (KCT), Knowledge of Content and Curriculum (KCC) and Knowledge of Content and Students (KCS). If this were the case the internal representations of those domains in the model would necessarily be larger to illustrate their importance (Figure 4). The content domains would be shown as being smaller to show that these were not the focus. However it would be inconceivable that these domains would have no impact whatsoever and therefore need to be included in the diagram. In a different context, if there was a perceived weakness in content knowledge and this was being addressed, then this diagram would obviously not be truly illustrative of the situation.

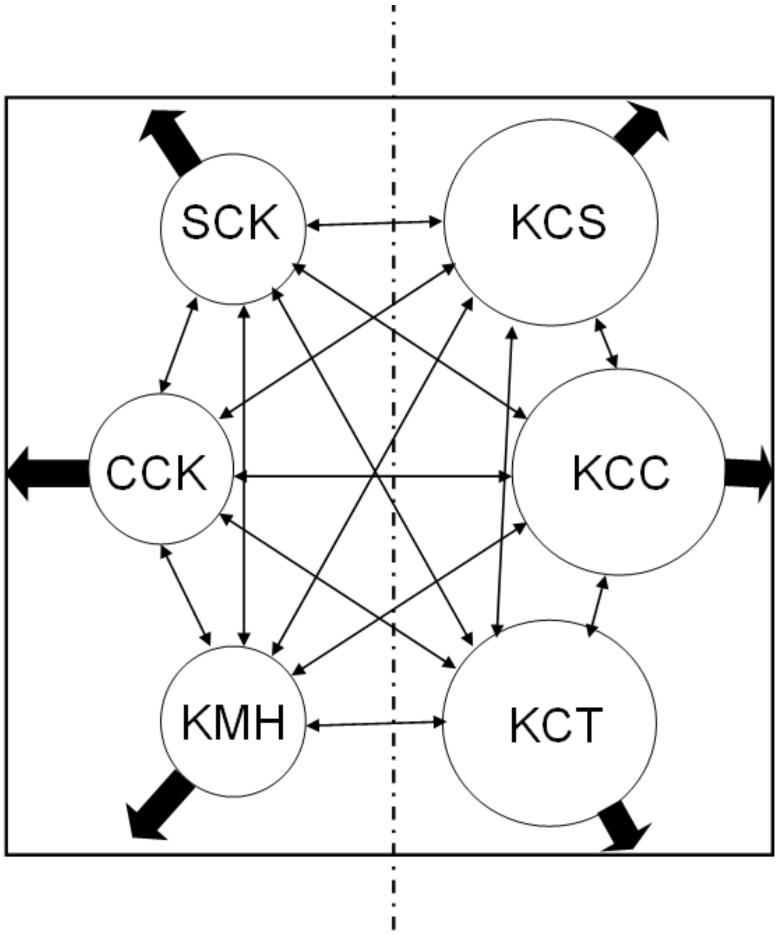

Figure 4 Domains of differentiated size to portray importance of Pedagogical Knowledge in PL

In the revised Mathematical Knowledge for Teaching model (Figure 3) the pedagogical domains are shown 'feeding' into an area designated as Pedagogical Knowledge. In the 2008 Hill et al. model (Figure 2) this field is treated as a heading and given the label of Pedagogical Content Knowledge (PCK). As previously stated, research would assert that PCK is actually the overlap of Subject Matter Knowledge (SMK) and Pedagogical Knowledge (PK) and that Figure 2 does not show this overlap. Rather it displays SMK and PCK as sitting as headings without reference to the required interconnectedness.

The final difference between the two models is in the use of arrows in the revised MKT model to illustrate the various interactions between the domains. The 2008 Hill et al. model (Figure 2) suggests that each of the domains is discrete and entire of itself. It could be argued that exercising any of the domains brings into play other domains, for example, it is not feasible in the context of teaching and learning to talk about content devoid of pedagogy or pedagogy devoid of content. Even those who teach by didactic chalk and talk are illustrating a pedagogy.

Nor does this interaction occur just across the two fields. Often there is interaction within the domains which feed a field. For example, instruction on Specialised Content Knowledge (SCK) will be built on Common Content Knowledge (CCK).

Conclusion

Teachers are quite capable of engaging in reflective practices regarding their teaching (Galae, 2012). To facilitate this reflection occurring, it may be beneficial for some to have a model from which to work and scaffold their understanding. PCK which was devised by

Shulman (1986) as a heuristic is certainly a consideration, but the Hill et al. (2008) construct with the modifications suggested in this writing may be a better place to start due to the development of Shulman's ideas. Hill and Charalambous's research (2012) supports that Mathematical Knowledge for Teaching (MKT) contributed to instructional quality, it therefore would not seem unreasonable to suggest that if we want to improve teacher effectiveness the development of MKT is an important factor. At the very least, familiarity with this construct would allow teachers to reflect on the various domains that require development to foster PCK, and allow them the opportunity to strengthen any areas in which they may feel they are deficient.

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